Synchronization of spatiotemporal chaos in asymmetrically coupled map lattices

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The synchronization of spatiotemporal chaos of two asymmetrically coupled map lattices is studied by numerical simulations. It is found that the synchronization can be achieved by linking two spatially extended systems with a common signal or signals through one-site connections. The synchronized states are found to be closely related to the approaches used to synchronize the most upstream sites of the two spatially extended systems. The effects of small backward diffusions and the local disturbances on the development of the synchronized states are also discussed. [S1063-651X(98)07404-2]

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The synchronization of chaotic systems has attracted considerable attention in recent years [1–11]. This synchronization has clear applications to communications, control and anticontrol of chaos in biomedical systems, and system identification. Furthermore, it may be responsible for the saturation of the invariant characteristics of chaos in chains of coupled nonlinear oscillators and in more complicated systems. It is likely that control and synchronization of chaos and hyperchaos play important roles in the workings of biological and artificial neural networks.

Currently the synchronization of hyperchaotic systems has become an active research area [12–17], due to its potential applications in secure communications. In Ref. [13], it is shown that two one-way coupled map lattices (OCML) [18–22] may be synchronized by linking the most upstream sites to a common chaotic signal, which is extracted from the same OCML system but with a periodic boundary condition, or to a common stochastic signal. It is also found that the spatially periodic and temporally chaotic states may be observed if the lattice size of the drive system is smaller than that of the response system. For other types of driving signals, no single-site-linking synchronization is reported.

In this paper, we study the synchronization of spatiotemporal chaos (STC) of two asymmetrically coupled map lattices (ACML) [23–25]. When the backward diffusion constants are set to zero, ACML reduces to OCML and so under certain conditions, it is expected that the ACML may behave in a similar way as the OCML does under certain circumstances. On the other hand, it has been shown that a nonvanishing backward diffusion may dramatically change the dynamical behavior of the system under consideration [26]. In view of the influence of the downstream sites through the presence of backward diffusion couplings in an ACML system, it seems impossible to expect that the synchronization of STC can be attained via one-site connection when the backward diffusion is appreciable. To understand the general features of the ACML systems, we performed extensive numerical simulation studies, and investigated the influence of the presence of the backward diffusion on the synchronization of spatially extended systems, which are linked through single-point coupling.

To demonstrate spatiotemporal synchronization of open flow systems, we consider the following ACML [22–25]:

$$x_{n+1}^{i}=(1-\gamma_1-\gamma_2)f(x_n^i)+\gamma_1 f(x_n^{i-1})+\gamma_2 f(x_n^{i+1}), \quad (1)$$

where x_n^i is the amplitude associated with the ith lattice site at time step n. We take the logistic map $f(x)=1-ax^2$ as the local element and choose the nonlinearity a to be well within the chaotic regime. We further assume that $\gamma_1>\gamma_2\geqslant 0$. When $\gamma_2=0$, Eq. (1) reduces to the one-way coupled logistic lattice (OCLL). For open flow systems, the boundary conditions will strongly influence the dynamical behavior of the system. We consider the following two different boundary conditions: (i) the periodic boundary condition $x_n^{i+N}=x_n^i$, and (ii) the open boundary condition defined by

$$x_{n+1}^{1}=(1-\gamma_2)f(x_n^1)+\gamma_2 f(x_n^2),$$

$$x_{n+1}^{N}=(1-\gamma_1)f(x_n^N)+\gamma_1 f(x_n^{N-1}).$$

The replica of the open flow system to be synchronized is given by

$$y_{n+1}^{i}=(1-\gamma_1-\gamma_2)f(y_n^i)+\gamma_1 f(y_n^{i-1})+\gamma_2 f(y_n^{i+1}), \quad (2)$$

which is driven away by some signal s_n derived from the drive system (1) or is taken from a stochastic variable. Equation (2) is, therefore, called the response system. Recently, many different approaches to synchronize chaotic and hyperchaotic systems have been proposed. Among them are the active-passive decomposition (APD), direct substitution, and feedback control approach. The success of these methods is based on whether all Lyapunov exponents of the response system could be made negative. Generally speaking, to synchronizing hyperchaotic or spatially extended systems, one needs distributed linkings or at least some dense lattice of connecting nodes. In many applications, however, it is very

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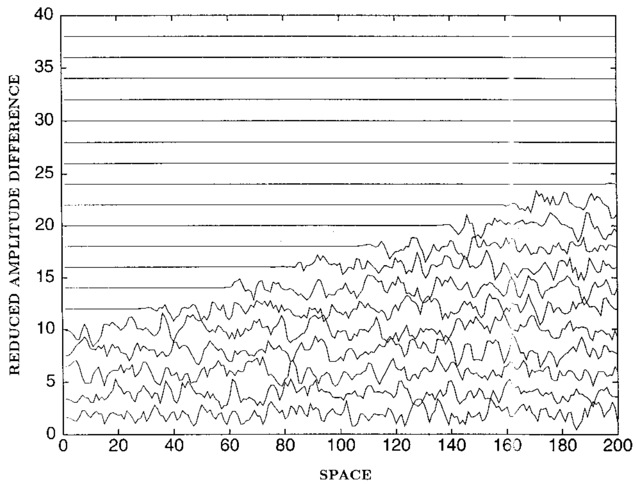

FIG. 1. The propagation of synchronization front. The reduced amplitude difference dx_n $(=y_n^i-x_n^i+n/200)$ is plotted against the space for every 400 iterations, starting from randomly chosen initial conditions. The system size is $N=200$, $a=1.9$, $\gamma_1=0.7$, and $\gamma_2=0.1$. The results are independent of the systems size and are qualitatively the same for different machines.

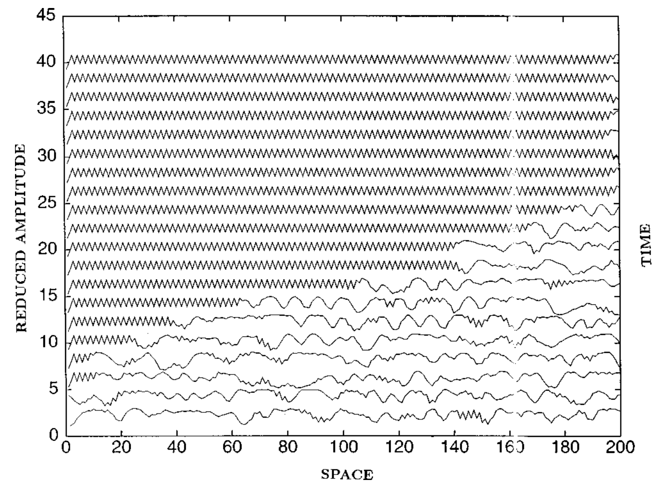

FIG. 2. The suppression of spatiotemporal chaos by applying the feedback control approach (3) to the first sites of two ACML's. The system parameters are taken to be $a=a'=2$, $\gamma_1=0.51$, and $\gamma_2=0.35$.

difficult to make a one-to-one contact between all elements of the drive and response systems. Therefore, it is of great theoretical and practical interest to search for the few-point-linking synchronization schemes that are feasible for spatially extended systems.

In this paper, we apply different synchronization approaches on the first sites of two ACML's. Since the boundary conditions play an important role in the development of spatiotemporal waves in open flow systems, we focus our attention on two cases: (A) The open boundary condition is used for both the drive and the response systems, (B) the periodic boundary condition for the drive system while the open boundary condition is assumed for the response system. It is interesting to note that the computer simulation results for open flow systems are often machine dependent, that is, for certain values of system parameters, different computers may yield qualitatively different behaviors. The results reported here are only those properties that are, at least, qualitatively the same for different machines. Our main results can be summarized as follows.

(A) Open boundary conditions for both the drive and the response system: In this case the sizes of both systems are assumed to be the same. We first couple the first sites of the two systems to a common stochastic variable, i.e., $x_n^1=\xi_n$ and $y_n^1=\xi_n$. It is found that for γ_2 fixed, there exists a critical value such that for $\gamma_1>\gamma_c$, two systems may be synchronized with each other. The same is true for connecting directly the first sites of the two systems, i.e., $x_n^1=y_n^1$. Generally speaking, the increase of γ_2 and a leads to the destabilization of the synchronized state, while the increase of γ_1 may favor the synchronization. Next, we tried to use a chaotic signal to drive the first sites. Here, we take that $x_n^1=z_n$ and $y_n^1=z_n$ with $z_{n+1}=1-a'z^2$. It is interesting to find that at $a=a'=2$, $\gamma_1=0.8$, the synchronization is observed for $\gamma_2=0.01$, but not for $\gamma_2=0$, which indicates that the backward diffusion may possibly reduce the noise effects [26]. As is expected, further increasing γ_2 will destroy the synchronization. Another interesting phenomenon is that if $a<a'$, the synchronization of spatiotemporal chaos may also be achieved. Nevertheless, if $a>a'$, no synchronization is observed.

We now turn to the feedback control scheme of chaos synchronization, which is defined as

$$x_{n+1}^1=(1-b)[(1-\gamma_1-\gamma_2)f(x_n^1)+\gamma_1 f(x_n^N)+\gamma_2 f(x_n^2)]+bz_n,$$

$$y_{n+1}^1=(1-b)[(1-\gamma_1-\gamma_2)f(y_n^1)+\gamma_1 f(y_n^N)+\gamma_2 f(y_n^2)]+bz_n, \tag{3}$$

where $z_{n+1}=1-a'z_n^2$ or $z_n=\xi_n$ with ξ_n being a random variable. It is expected that this kind of coupling may provide a much stronger synchronization forcing because once the first sites are synchronized with the chaotic driving or

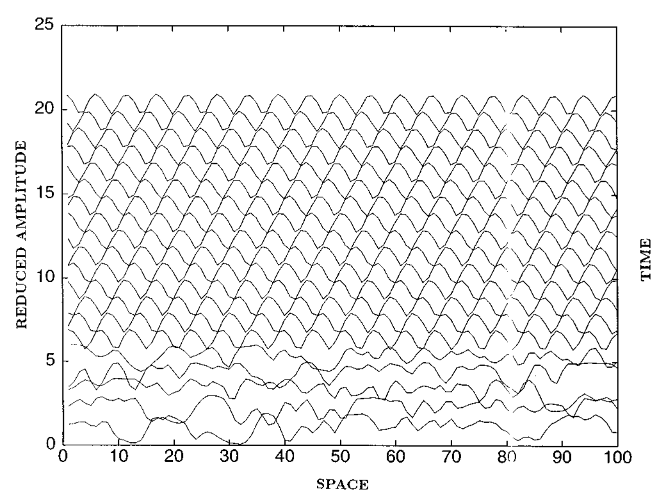

FIG. 3. The traveling wave in a synchronized state at $a=2$, $\gamma_1=0.51$, $\gamma_2=0.1$, $N_d=22$, and $N_r=100$. The reduced amplitude $Y_n^i=y_n^i+n/1000$ is plotted against the space i at every 2000 steps.

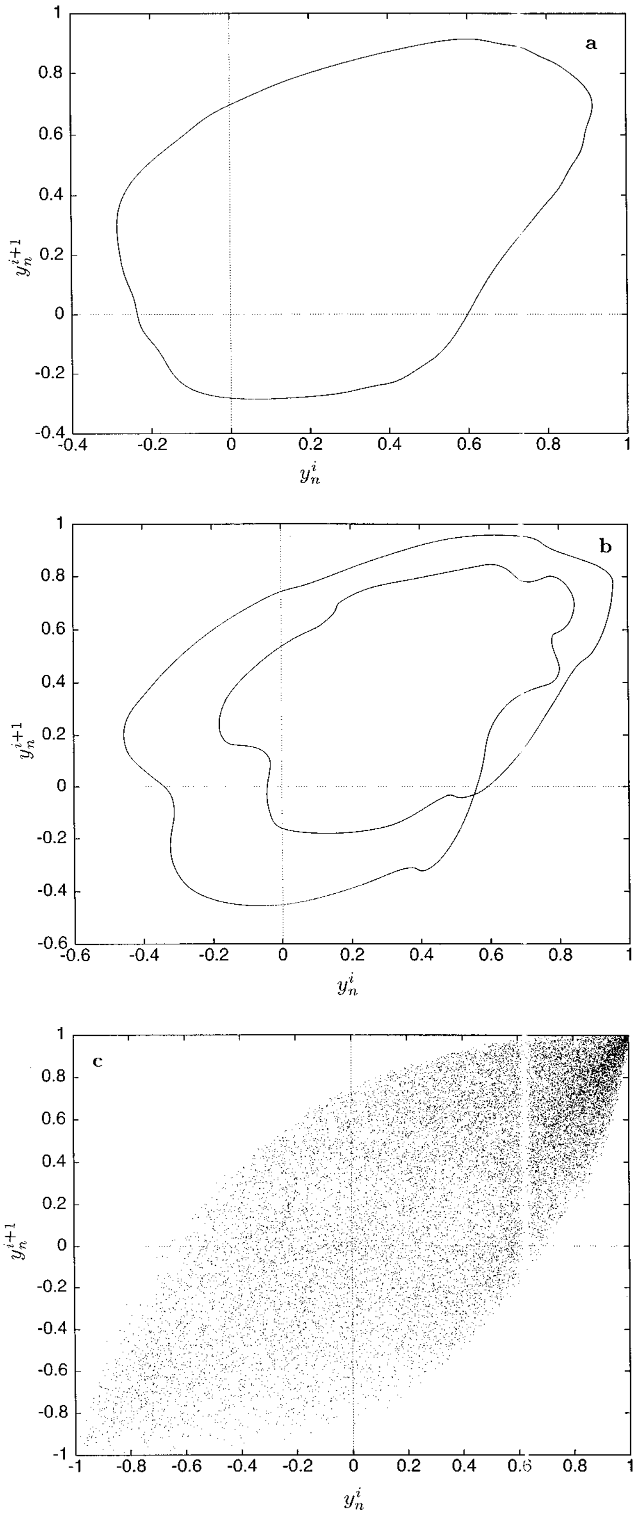

FIG. 4. The return map (y_n^{i+1}, y_n^i) for $i=31$. The system parameters are the same as in Fig. 3, except that the size of the drive system is taken to be (a) $N_d=22$, (b) $N_d=23$, and (c) $N_d=24$.

stochastic driving, the attractor traced out by the first sites will be nonchaotic, which implies that the nearby trajectories will converge instead. We found that the synchronization can be achieved even for moderate backward diffusions. Figure 1 shows a typical evolution of synchronization wave, starting from the most upstream edge. It is remarkable that this feed-

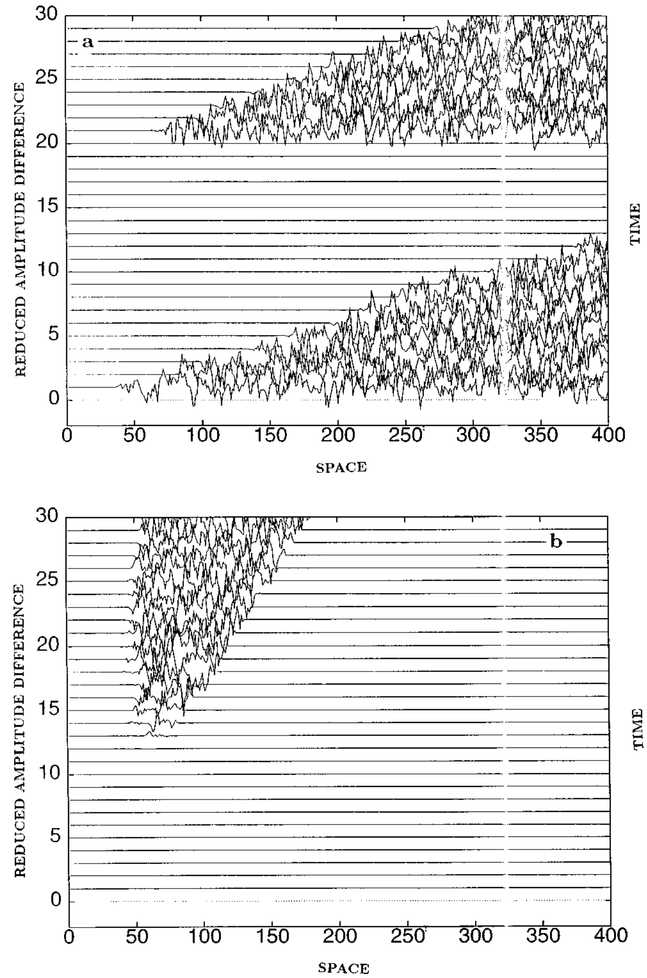

FIG. 5. (a) The evolution of the initial pointlike disturbance of small amplitude at the left-hand boundary in a spatially uniform state. The system parameters are $N=400$, $a=2$, $\gamma_1=0.72$, and $\gamma_2=0.1$. The amplitude of the pointlike disturbance is $\delta=10^{-9}$. Note that the connection between two systems is cut off between the step 7200 and 8400. The local small perturbation is added at the step 8000. (b) The incipient development of the disturbance is shown every 6 iterations after the quenched perturbation is switched on at the iteration 8000.

back control approach may also result in the suppression of spatiotemporal chaos through one-site control. Figure 2 shows the spatial period-2 and temporal period-1 state obtained by controlling the first site of the ACML, using Eq. (3). It is worthwhile to point out that even for not very highly asymmetrical couplings, the one-point control method still works.

We also studied the synchronization via mutual connections, or bidirectional couplings. In our case, we consider the following synchronization method:

$$\begin{aligned} x_{n+1}^1 &= (1-b)[(1-\gamma_1-\gamma_2)f(x_n^1)+\gamma_1 f(x_n^N)+\gamma_2 f(x_n^2)] \\ &\quad + b f(y_n^1), \\ y_{n+1}^1 &= (1-b)[(1-\gamma_1-\gamma_2)f(y_n^1)+\gamma_1 f(y_n^N)+\gamma_2 f(y_n^2)] \\ &\quad + b f(x_n^1). \end{aligned} \tag{4}$$

We found that the synchronization of spatiotemporal chaos can be achieved for moderate values of the linking constant b. At $a=2$, $\gamma_1=0.8$, $\gamma_2=0.05$, and $b=0.8$, we found that the synchronized state is spatiotemporally periodic, indicating the suppression of spatiotemporal chaos through one-point mutual couplings.

(B) The periodic drive system versus open response system: We consider only the synchronization of the drive system with periodic boundary condition and the response system with open boundary condition via the direct substitution method, i.e., $x_n^1=y_n^1$. In general, if the drive and response systems are of the same size, we found similar behavior as in case (A). If, however, the drive and the response systems have different sizes, the synchronized states will vary. For instance, if N_d (the size of the drive system) is greater than N_r (the size of the response system), then all elements of the response system may be synchronized with those of the drive system, except a small number of sites near the downstream edges. On the other hand, if $N_d<N_r$, then we may observe different behaviors, depending on the boundary conditions used by the drive and response systems. When the periodic boundary condition is used in the drive system, the response system will exhibit temporally chaotic but spatially periodic patterns with period N_d. Depending on the size of the drive system, the system may show stationary, or traveling patterns with irregular or relatively regular wave forms. By inspection of the attractor of the return maps, one finds periodic doubling bifurcation as the size of the drive system varies (see Fig. 3).

Since our model system is convective unstable, arbitrary weak noise destroys the synchronized state. So it seems interesting to study the effects of the external perturbations on the synchronization of spatiotemporal chaos. Mathematically, when two systems are completely synchronized, they will remain synchronized even if the synchronizing coupling is removed. In Fig. 4 we show the effect of quenched local disturbance on the synchronized state. First we use the direct substitution method at a single site to synchronize two hyperchaotic systems. When we turn off the synchronizing coupling, it is seen that the two systems remained synchronized. At some later time, we add a perturbation at the same site that is used for passing the synchronization signal, for a very short duration (a few iterations), and then we impose the synchronizing force again. We found that the synchronized state after removal of the synchronization linking is unstable to the weak noise, as is expected. The synchronizing force also plays a role of control, which makes the system stable against external noise.

Another interesting feature found for the asymmetrically coupled map lattices is the generalized synchronization in which the synchronization relationship is of the form $y=\phi(\mathbf{x}(t))$. For concreteness, we take $y_n^i=x_{n-k}^{i-j}$, which is called the space-shift and time-delay synchronization. The synchronization is found for periodic boundary condition for both the drive and the response systems with equal size. This finding may be relevant for manipulating the message in the secure communications (see Fig. 5).

In conclusion, we have shown that synchronizing spatiotemporal chaos in ACML systems may be achieved by using one-site linking between the drive and the response systems. We have investigated the influence of the backward diffusion on the development of the spatiotemporal states. It is interesting to find that different synchronization schemes may give rise to different dynamical behaviors of the ACML system, which marks the difference between the ACML and the OCLL. It is clear that if there is no feedback to the first site from its neighbor ($\gamma_2=0$), then the synchronized state would be independent of the synchronization approach used. Our numerical results reveal that the conclusions drawn from the study of the OCLL systems are generally not applicable to open flow systems. In all our simulations carried out on coupled map lattices, we noted that the properties of the dynamical system may be qualitatively different by using different machines. Such a difference becomes appreciable when the system under consideration is highly asymmetrically coupled, such as in OCLL. This problem might persist in other spatially extended systems with asymmetrical couplings.

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